Molecular dynamics simulation of strongly coupled QCD plasmas

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The properties of a strongly interacting quark plasma are investigated by molecular dynamics method including non-abelian quark-quark potential. Our main goal is to study the thermalization process in this system. We find an interesting resonance-like behaviour: at a characteristic time close to the inverse plasma frequency the quark plasma is heated up substantially via energy transfer from quark potential energy into one particle kinetic energy. Color rotation mechanism enhances the effectivity of this heating process, leading to a very fast thermalization with high temperature.

1. Introduction

Early theoretical speculations suggested the appearance of a weakly interacting perturbative quark-gluon plasma state in heavy ion collisions at ultrarelativistic energies. However, recent data obtained at RHIC experiments, and their theoretical analysis, indicate the formation of a strongly interacting deconfined matter [1]. In fact, lattice-QCD calculations on equation of state have already shown the strong coupling and the theoretical analysis based on massive quasi-particle picture described quantitatively many properties of this matter [2]. Thus we expect a relatively dilute, massive, quark and antiquark dominated matter to appear before hadronization. Widely used molecular dynamics simulation [3, 4] is the ideal tool to study such a particle compound, to investigate non-perturbative features and to determine different properties in the strongly interacting matter. In this simulation we can study the influence of the color charge fluctuation and pair correlation function can be generated, displaying if the quark matter is gaseous or rather liquid like. Although molecular dynamics models for non-abelian interactions are very much approximate, we can study qualitatively very important microscopic processes, for what other methods do not exist. Here we summarize the main findings of our work, details can be found in Ref. [5].

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2. The Molecular Dynamics Model

In the model the quarks are the only actively simulated species, the gluons are represented as a background field. This approach is similar to the case of the classical multicomponent or one component plasma (OCP) model, where the motion of the ions is of interest, the electronic background is treated as a constant field [4, 6].

In the model pairwise interaction of the quarks is assumed. The possible realizations of (single flavored) two-quark (QQ) system are composed of symmetric sextet and antisymmetric anti-triplet combinations.

The quark-quark effective interaction is represented by a Coulomb-like potential in the form of

$$V = \langle \lambda_i \cdot \lambda_j \rangle \frac{\alpha}{r_{ij}} \qquad (1)$$

acting between the i^{th} and j^{th} particle, where $\alpha = g^2/4\pi$ is the interaction coupling constant, r_{ij} is the inter-particle distance and $\langle \lambda_i \cdot \lambda_j \rangle$ depends on the color states of the particles as shown in Table 1. The parameter D introduced in Table 1 represents values of $\langle \lambda_i \cdot \lambda_j \rangle = +\frac{1}{3}$ or $-\frac{2}{3}$ randomly chosen with equal probabilities, due to the random realization of symmetric and antisymmetric QQ-states for all different colored pairs. This has the effect, that equally colored quarks interact always repulsively, but different colors may attract or repell each other. An additional short-distance cutoff in the interaction is introduced: at $r < 0.1$ fm the potential is taken as a linear function of r, which results in a smooth quadratic decrease of the inter-particle force for $r \to 0$ (soft-core model). This arbitrary cutoff can be justified in view of the finite spatial extension of the quark wave-function.

Table 1: $\langle \lambda_i \cdot \lambda_j \rangle$ values for QQ-pairs.

	$\|R\rangle$	$\|G\rangle$	$\|B\rangle$
$\|R\rangle$	$+\frac{1}{3}$	D	D
$\|G\rangle$	D	$+\frac{1}{3}$	D
$\|B\rangle$	D	D	$+\frac{1}{3}$

The bookkeeping of all possible QQ pair-interactions in the system is realized using the so called "interaction table" of which an example is shown in table 2. The particles are assigned a label and all particles have an associated color value. The elements of the table show the $\langle \lambda_i \cdot \lambda_j \rangle$ values of the actual QQ pair realizations. The values of the D entries are randomly redistributed periodically (see below). The total system is color-balanced (white), but neglecting the self-interaction results in the appearance of a net attraction.

Table 2: Interaction table example for a 9-quark system. Dark and light gray fields are excluded due to the neglected self interaction and double counting, respectively.

	1_R	2_G	3_B	4_G	5_R	6_R	7_B	8_G	9_B
1_R		D	D	D	$+\frac{1}{3}$	$+\frac{1}{3}$	D	D	D
2_G			D	$+\frac{1}{3}$	D	D	D	$+\frac{1}{3}$	D
3_B				D	D	D	$+\frac{1}{3}$	D	$+\frac{1}{3}$
4_G					D	D	D	$+\frac{1}{3}$	D
5_R						$+\frac{1}{3}$	D	D	D
6_R							D	D	D
7_B								D	$+\frac{1}{3}$
8_G									D
9_B									

In the molecular dynamics simulation the quarks interact with the color-dependent effective pair-potential given by (1). Periodic boundary conditions are applied together with the Ewald summation technique for the proper treatment of the long range interaction. Forces acting on each of the particles (quarks) due to all other particles are calculated in every timestep and the equation of motion for each particle is integrated in time [3].

At the start of the simulation the particles are randomly distributed in the cubic simulation box with kinetic energies determined by an initial temperature. An initialization phase is used before any measurement is taken on the system. In this phase the momenta of the particles are scaled back according to the given initial temperature in every timestep. The simulation time of this phase is chosen to be long enough to reach equilibrium conditions for the unperturbed system over the subsequent measurement period.

The effect of the gluon field on the quark component is taken into account in three ways: (i) it represents an infinite heat reservoir, energy can be coupled out of it without any changes in the qluon field; (ii) gluons can transfer color charge, the color state of two quarks may be exchanged (color rotation) with a characteristic time given by τ_C; (iii) qluons perturb QQ-pairs, leading to a new realizations of the pair-interaction (redistribution of the interaction table) with a characteristic time τ_D.

The input parameters in our simulations are: density $n = 10$ quarks/fm^3, initial temperature $T_0 = 200$ MeV, effective quark mass $m_q = 300$ MeV/c^2, interaction coupling strength $\alpha = 1$, time between reassignment of all D (different-color pair elements) in the interaction table τ_D, time between color rotation events τ_C and the ratio of pairs color rotated in one event N_C. The plasma frequency is derived as $\omega_P^2 = 4\pi\alpha n/m_q$. The behavior of the system is now governed by the two, independently chosen "refreshment" times τ_D and τ_C.

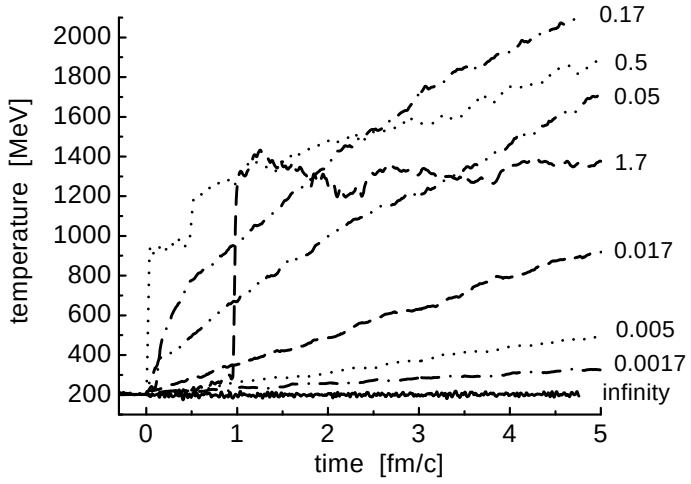

Figure 1: Time-evolution of the kinetic temperature for systems with different τ_D values at $\tau_C = 0$. (Numbers labeling the curves are the corresponding τ_D values in fm/c units.)

Figure 1 shows the time-evolution of the quark kinetic temperature for different τ_D values. It can be seen, that in the limiting cases $\tau_D = \infty$ and $\tau_D \to 0$ the temperature fluctuates closely to its initial value. Intermediate values of τ_D result in increasing temperatures.

The τ_D-dependence of the temperature is plotted in figure 2(a) for three snapshots in time. A remarkable resonant-like behavior of the heating of the system can be observed, where the maximum heating-rate establishes at τ_D values around the inverse plasma frequency $1/\omega_P \approx 0.19$ fm/c. Figure 2(b) shows the dependence of the resonant-like peak on an additional color rotation rate, where $\tau_C = \tau_D$. We have found that the color exchange increases the energy transfer from the background field to the quark component by 10 to 20%.

Figure 3(a) displays the pair-correlation function (PCF) for the $\tau_D = 0.0005$ fm/c case neglecting color-rotation. The total PCF is decomposed into contributions of equal-colored-, and different-colored pairs. The attractive force acting between some of the different-colored pairs result in the appearance of the correlation peak at $r \approx 0$. This indicates the onset of clusterization [7, 8] of particles which gets more pronounced at

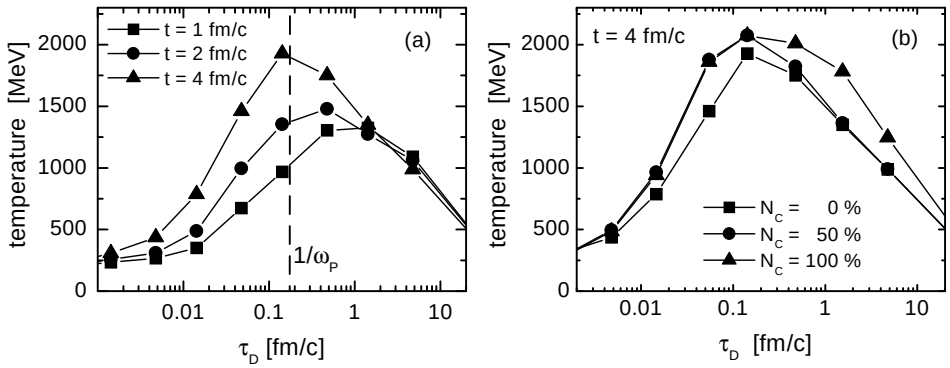

Figure 2. Temperature vs. τ_D at (a) times $t = 1$, 2 and 4 fm/c without color rotation; and (b) at time $t = 4$ fm/c with $\tau_C = \tau_D$ for color-rotation ratios $N_C = 0$, 50 and 100%.

higher τ_D values. Focusing only on the (always) repulsive equal-colored pairs, the PCF can be compared with classical OCP results [see figure 3(b)]. This comparison shows that the repulsive equal-colored component of the QQ plasma is structurally similar to classical OCP in the so called "gas" phase (at Γ values around unity), which indicates already strong non-ideality of the QQ-plasma investigated here.

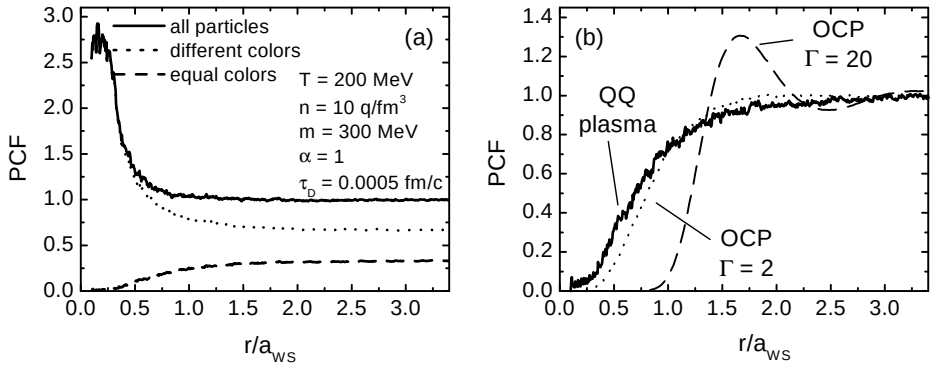

Figure 3. Pair-correlation function for the $\tau_D = 0.0005$ fm/c case (a) decomposed into equal-colored and different-colored contributions; and (b) compared to classical OCP plasmas. Here $a_{WS} = 0.3628$ fm is the Wigner-Seitz radius, and Γ is the plasma (non-ideality) parameter.

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